Fast Estimation of Dose Distribution in water generated by Electron Beam

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1. Introduction

About PITZ

Photo Injector Test Facility at DESY in Zeuthen

- PITZ develops optimized electron sources (minimized emittance) for short-wavelength Free Electron Laser (FEL) user facilities like the **European X-ray Free Electron Lasers** in Hamburg.
- R & D in the application of its high-brightness beam: THz FEL, **FLASH RT**

DESY has two sites:

Hamburg
https://images.app.goo.gl/9ymsdE5wWUcVR8Y37

Zeuthen
https://images.app.goo.gl/cpiB3DBMdQ5BYHGJ8

1. Introduction

FLASH-RT

- The Main Motivation: FLASH radiotherapy (RT) is a technique involving the delivery of high dose rate radiation to the target, that is **sparing of healthy tissue** by radiation with **short, high intensity pulses** (e, p, ion, x-ray, **> 40 Gy/s**) while having at least the **same tumor control** as with conventional radiation(uses x-ray)
- A startup beamline has been in operation since November 2022
  - Dosimetry (measurement and **simulation**)
  - *In vitro* experiments

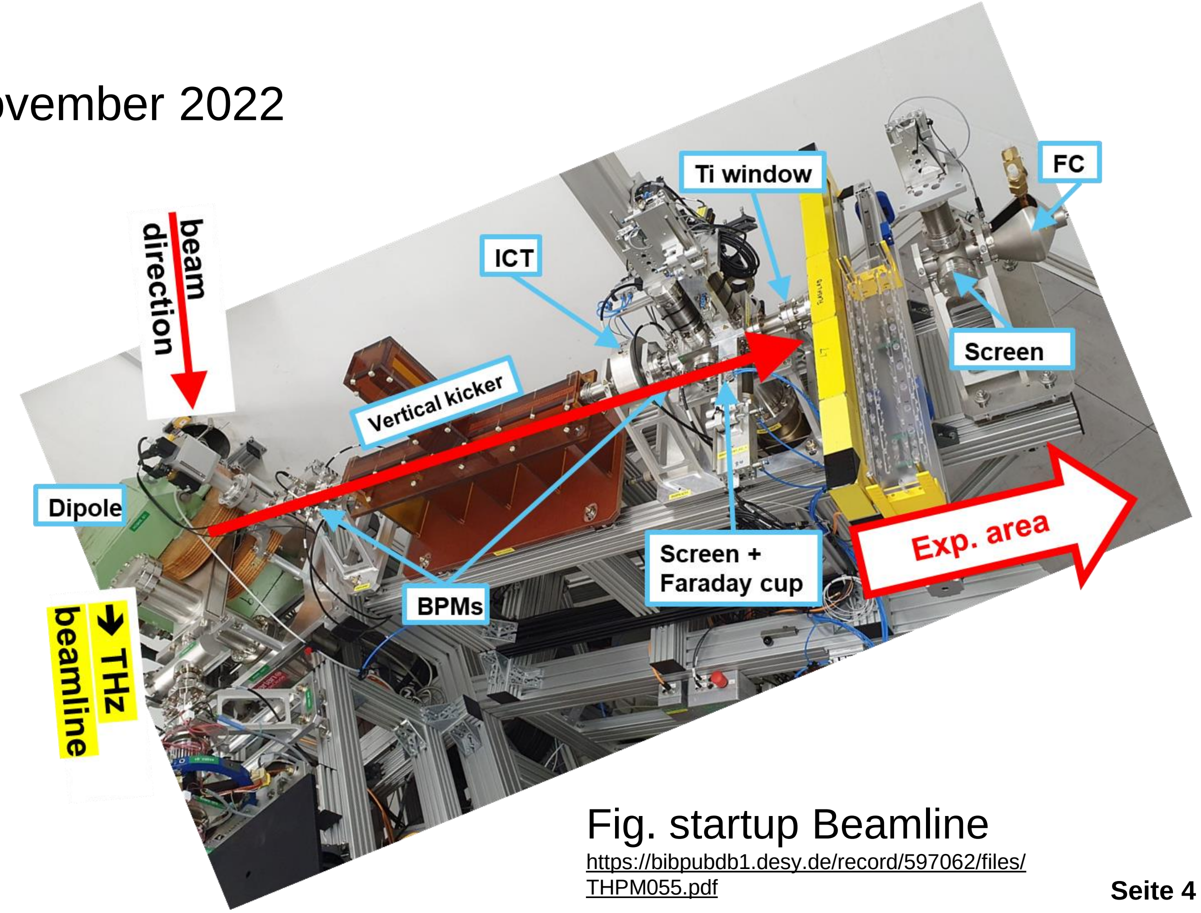


Fig. startup Beamline
https://bibpubdb1.desy.de/record/597062/files/THPM055.pdf

1. Introduction

About this work

- A Python script has been developed for the **FAST** estimation of Dose distribution in water by electron beam, based on Moliere Theory.
- This can be potentially used for online dose determination or first order treatment plan
- The script has been compared with FLUKA Monte Carlo simulation

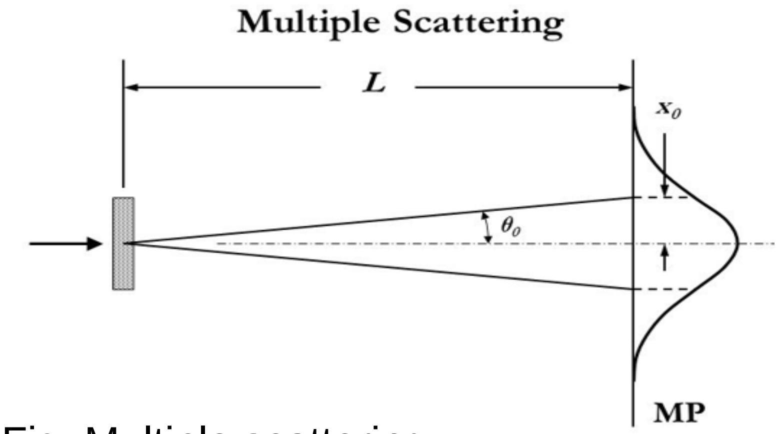


Fig. Multiple scattering

https://gray.mgh.harvard.edu/attachments/article/337/Techniques%20of%20Proton%20Radiotherapy%20(06)%20Multiple%20Scattering.pdf

- Monte Carlo simulation is a method from the Probability theory, in which random samples of distribution are repeatedly drawn using random experiments.
- Monte Carlo simulations are particularly suitable for calculating the expected value of a function, but usually takes long time

The Molière theory of multiple scattering is based on the standard transport equation, the Bessel transforms and the small angle approximation.

2. Fast Estimation of Dose Distribution in water

- Which effects we considered
  - Energy loss → change of Energy in water
  - Multiple scattering from collision → the rms scattering angle values(χ_c^2)
  - Electron screening of Coulomb potential because of atomic nucleus → $\chi_\propto^2$
  - Lateral displacement → the rms transverse displacement (y_M)

- How to calculate the spatial distribution
  - The angle distribution is given by Moliere theory
  - The spatial distribution is scaled to the angle distribution

https://iopscience.iop.org/article/10.1088/0031-9155/61/4/N102/pdf

2.1 Energy loss in water

Stopping Power

- **Stopping Power** – When charged particles interact at low energies, it describes the energy that is lost **.** The ability of a substance to slow down energetic particles moving through its interior is measured by its stopping power

- Unit – MeV/cm^2g^{-1}

- $$-\frac{1}{\rho}\frac{dE}{dz} = S(E)$$

dE – change in energy

dz – change in distance

S(E) – Stopping power dependent on energy.

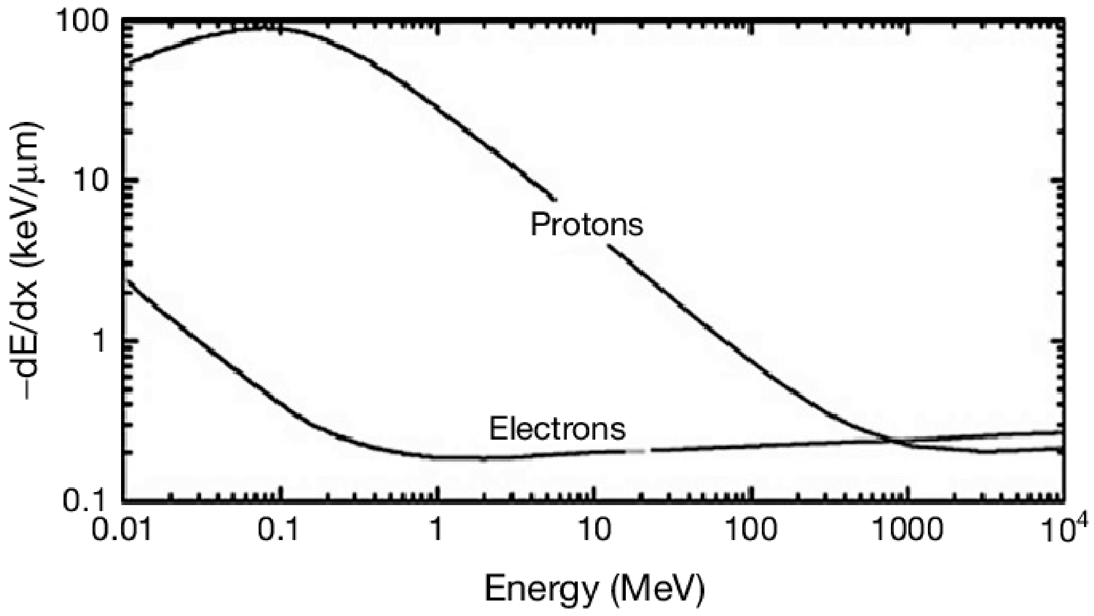


Fig. Stopping power Vs Energy Alloni et al. Early Events Leading to Radiation-Induced Biological Effects. In: Anders Brahme,editor-in-chief. Comprehensive Biomedical Physics, Vol 7, Amsterdam:Elsevier; 2014. p. 1-22. In Press

2.1 Energy loss in water

Calculation of Energy loss by using Stopping Power

- Energy loss is calculated by using stopping power with the help of Runge-kutta method in medium water by using Python script
    - The most widely known member of the Runge-Kutta family is generally referred to as “RK4”

$$-\frac{1}{\rho}\frac{dE}{dz} = S(E)\ , E(z0) = \ E0$$

$$\rightarrow E \sim z$$

- For each step of h, four coefficients are calcuated first

$$k_1 = f\,(z_n,\ E_n\ )$$

$$k_2 = f\,(z_n * h/2,\ E_n + h*K_2/2)$$

$$k_3 = f\,(z_n + h/2,\ E_n + h*k2/2)$$

$$k_4 = f\,(z_n + h,\ E_n + h*k3)$$

- Then the distance and energy are updated:

$$z_{n+1} = z_n + h$$

$$E_{n+1} = E_n + h/6*(k_1 + k_2 + k_3 + k_4)z_n$$

- From the energy, momentum (p(z)) and beta($\beta(z)$) are calculated along the distance z in water
    - momentum and beta are needed for calculation of RMS scattering angle, electron screening of the Coulomb potential, RMS transverse displacement

2.1 Energy loss in water

Comparison to online data

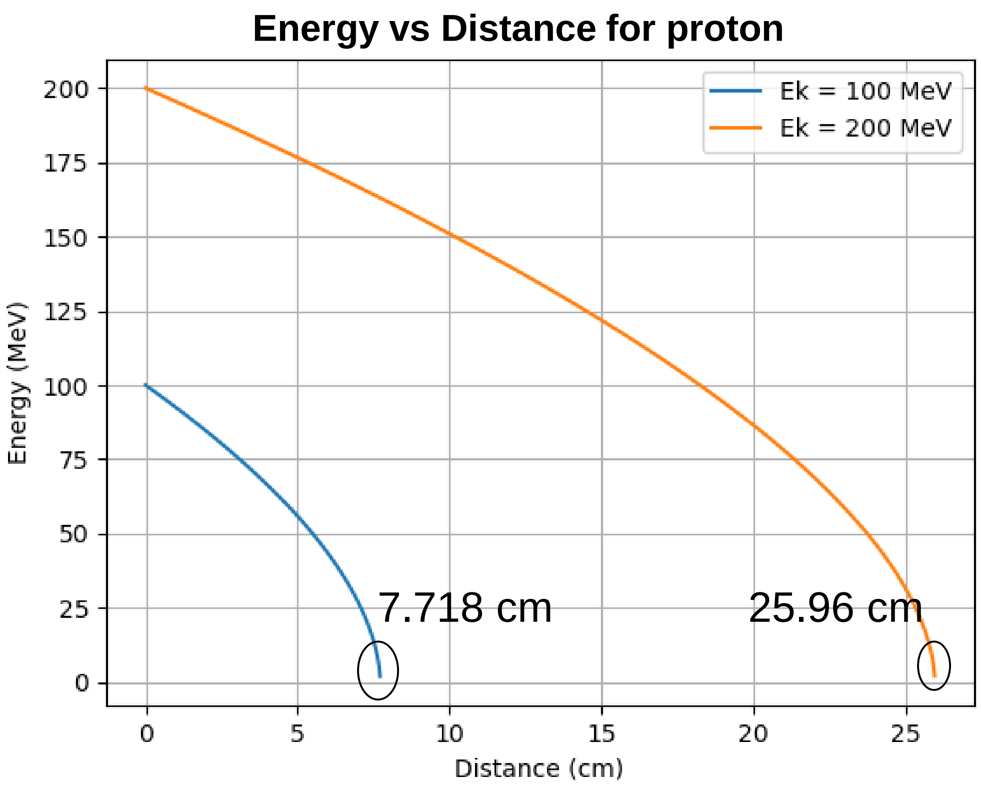


Fig. Comparison of energy loss at 2 different energies in case of proton

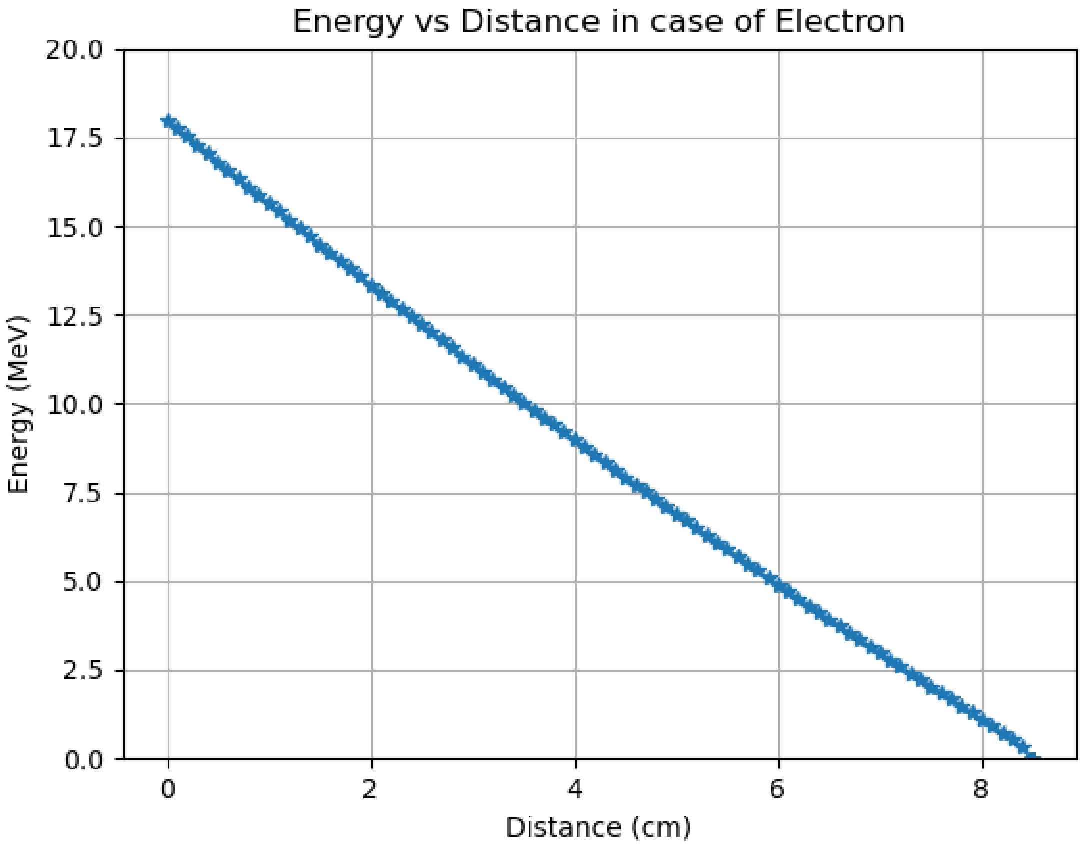


Fig. Simulation of energy loss in case of electron

Data = https://physics.nist.gov/cgi-bin/Star/ap_table.pl

2.2 Calculation of RMS scattering angle

- χ_c^2 is the rms scattering angle value that is calculated from the following formula –

$$\chi_c^2(z) \; = \chi_w^2 \int_0^z \frac{1}{p(t)^2 \beta(t)^2} \mathrm{dt}$$

- Where χ_w^2 = 0.1569*$10^{-6} \frac{{z_e}^2(2Z_H^2+Z_O^2)}{A_M}$, this part represents the energy loss independent part in case of water
  - Z_i is atomic number of an atom (for H_2O, Z_H, Z_O = 1, 8 respectively), z_e is atomic number of incident particle (in case of electron it is one), A_M is atomic mass of an atom
- $P(z)$ is the momentum that depends on distance (z), $\beta(z)$ is velocity of incident particle that varies with (z), which are calculated from the energy ($E(z)$ in 2.1)

2.3 Calculation of the electron screening of Coulomb potential

- $\chi_{\propto}^2$ is describing here the electron screening of the Coulomb potential.

$$\ln \chi_{\alpha}^2 (z) = \frac{1}{\chi_c^2(z)} \frac{0.1569.10^{-6} z_e^2}{A_M} \sum n_i Z_i{}^2 \int_0^z \frac{\ln \mu_i^2 \chi_{0i}^2 - \frac{D_i}{Z_i}}{p(t)^2 \beta(t)^2} dt$$

- Here

$$\mu_i^2 = 1.13 + 3.76 \frac{z_e^2 Z_i{}^2}{137^2 \beta(t)^2}$$ is the function of β(t)

$$\chi_{0i}^2 = \left(\frac{\hbar}{p(t)} \frac{Z_i^{\frac{1}{3}}}{0.468.10^{-8}} \right)^2$$ or $4.216 \,.\, 10^{-6} \frac{Z_i{}^{1/3}}{p(t)}$ is the function of p(t)

$$D_i = \ln \frac{1130}{Z_i{}^{4/3}\left(\frac{1}{\beta(t)^2} - 1\right)} + u_i - \frac{\beta(t)^2}{2}$$ is the Fano correction

$u_i = u_H , u_O$ in case of water u_H=3.6 , u_O = 5.0

Simulated diagram of χ_c^2 **and** $\chi_\propto^2$

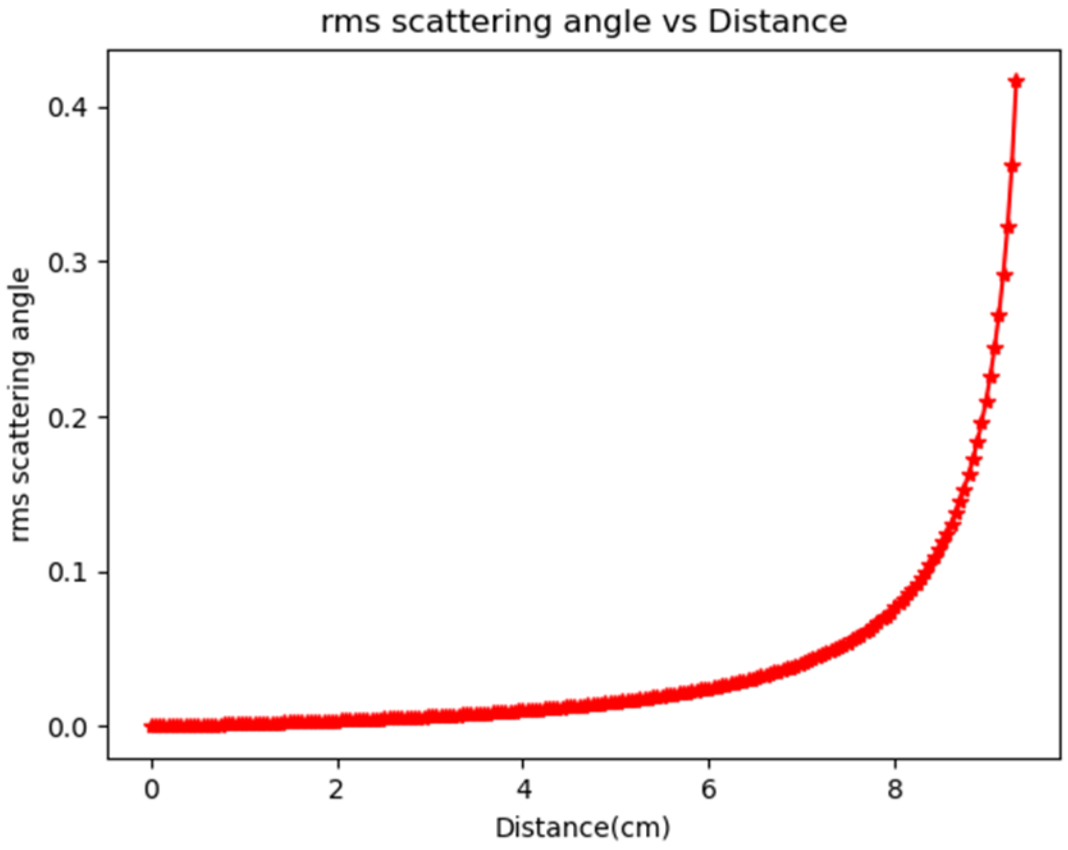


Fig. χ_c^2 vs z(cm)

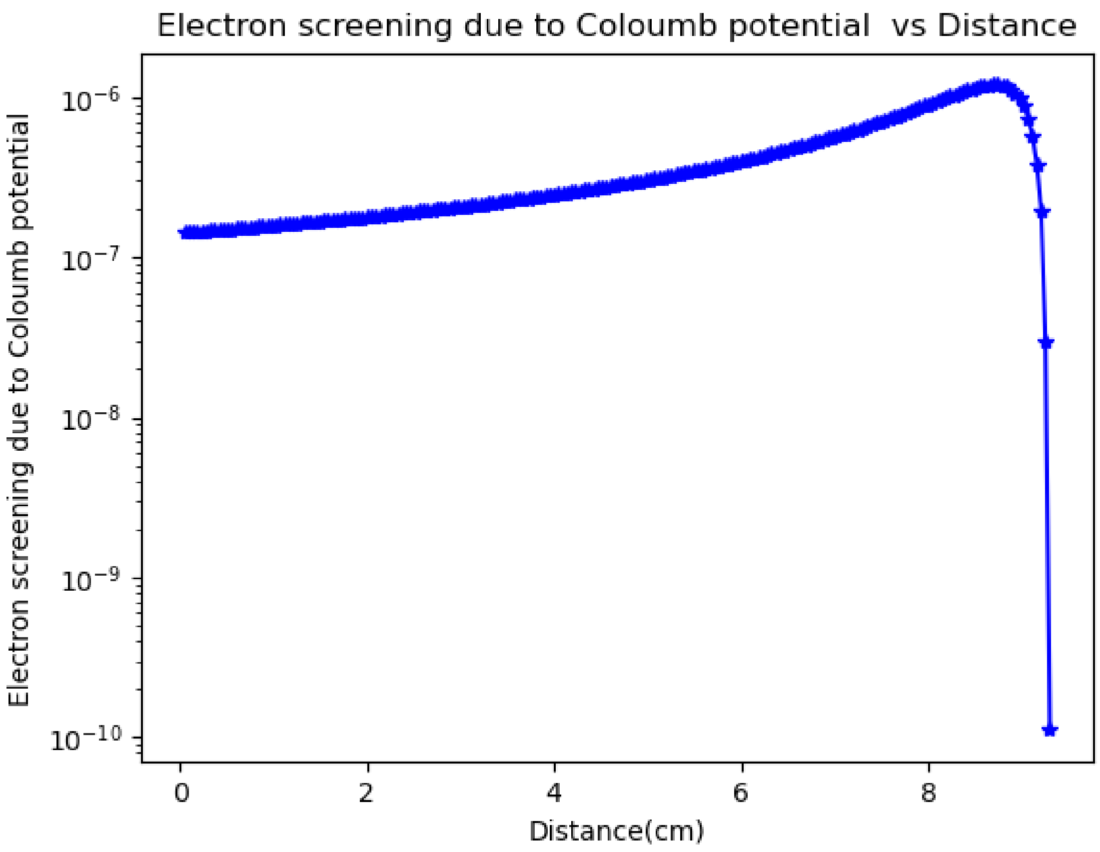


Fig. $\chi_\propto^2$ vs z(cm)

2.4 Calculation of lateral distribution

RMS of lateral distribution

- y_M is the rms lateral displacement on a measuring plane at z due to a layer dt at the depth t

$$y_M^2(z) = \frac{\chi_w^2 B}{2} \int_0^z \frac{(D-t)^2}{p(t)^2 \beta(t)^2} dt$$

- Here,

D is the distance of detector plane from medium,

t is depth of layer,

z is distance travelled in medium or thickness,

$\Omega_0 = \frac{\chi_c^2}{\chi_\alpha^2}$, is the total number of multiple scattering events

B = 1.153 + 1.122lnΩ_0

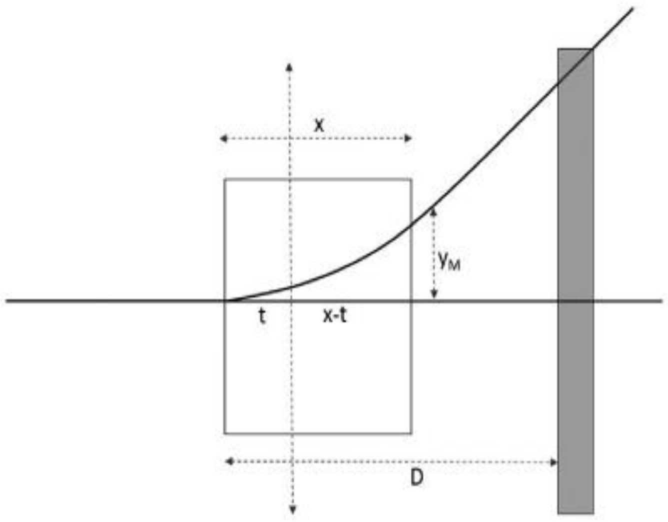


Fig. The geometry of the lateral displacement.
https://iopscience.iop.org/article/10.1088/0031-9155/61/4/N102

2.4 Calculation of lateral distribution

RMS of lateral distribution

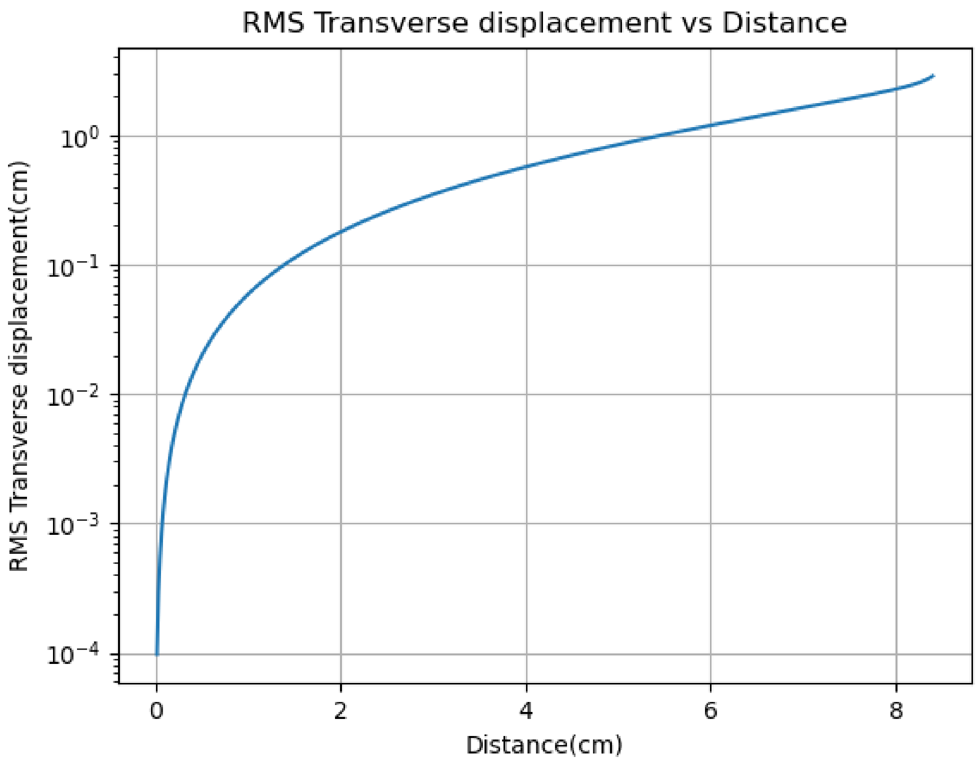


Fig. RMS Transverse displacement vs Distance

2.4 Calculation of lateral distribution

Scale from angular to spatial distribution

The rms from 2.4 (y_M) corresponds to the projected angle θ_R(z), which follows the standard form of Moliere distribution given by Scott(1963),

$$f(\theta)\theta d\theta = \frac{\theta d\theta}{\chi_c^2}\int_0^{\Gamma} \mathrm{J}_0\frac{\theta\eta}{\chi_c}\exp\left[-\frac{\eta^2}{4}\left(b-\ln\frac{\eta^2}{4}\right)\right]\eta d\theta$$

Here,

b = $\ln\Omega_0 \; - 0.154432,$

J_0 is Bessel function, $b-\ln\frac{\eta^2}{4}$

Γ = 2exp$\left[\left(\frac{b-1}{2}\right)\right]$, that has to be chosen at minimum of the exponent in the integrand.

The factor $\delta = \frac{y_M}{\theta_{zR}} = \frac{y_M\sqrt{2}}{\chi_c\sqrt{B}}$ represents the scale factor from angular to spatial distribution, where, $\theta_R(z) = \chi_c\sqrt{B},$ is rms of the Gaussian core of the angular distribution.

2.4 Calculation of lateral distribution

Scale from angular to spatial distribution

Replacing the variable in the prev. equation with

$$\delta = \frac{y}{\theta_z} \longrightarrow \theta_z = \frac{y}{\delta}$$

We get the spatial distribution

$$f_M(y) = \frac{1}{\pi\chi_c\delta}\int_0^{\Gamma} \mathrm{J}_0\left(\frac{y\eta}{\chi_c\delta}\right)\exp\left[-\frac{\eta^2}{4}\left(b - \ln\frac{\eta^2}{4}\right)\right]\eta d\eta$$

2.4 Calculation of lateral distribution

Comparison between Molière and Gaussian Distribution

- Wider tail in Molière distribution
- At mean position have similar distributions

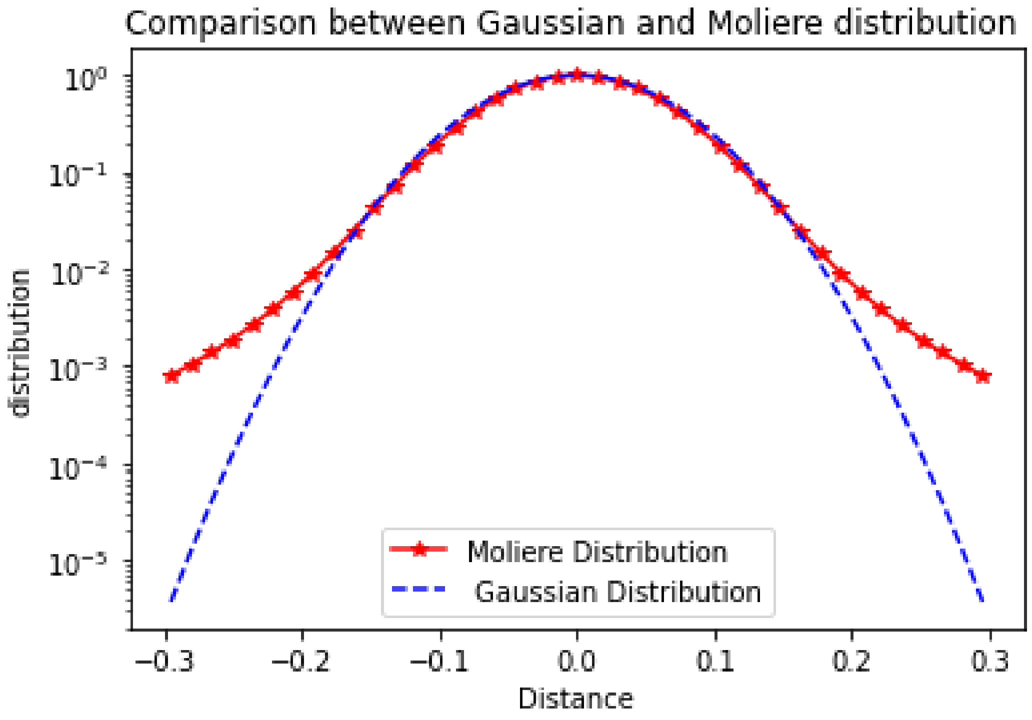
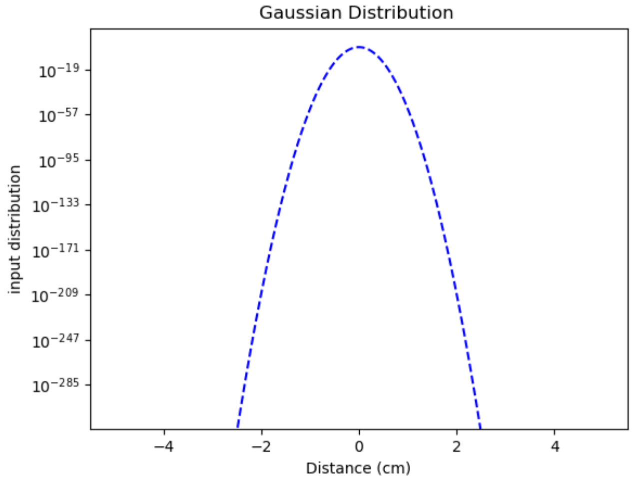


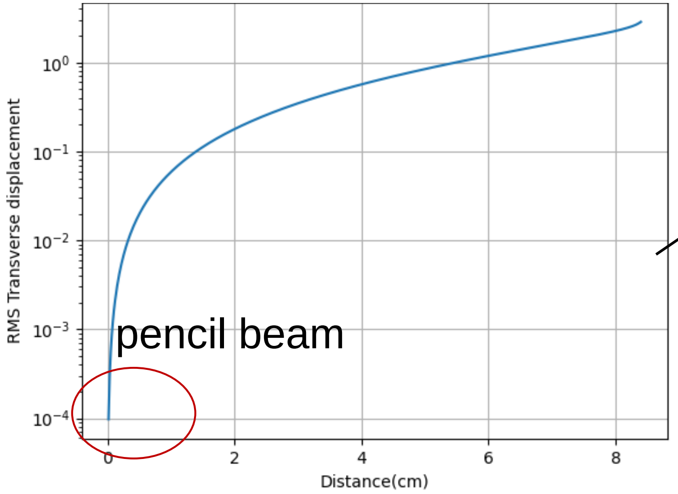
Fig. Comparison of Distributions

2.5 Consider a real beam with limited size

Why convolution is needed?

- The real beam has a distribution at the water entrance





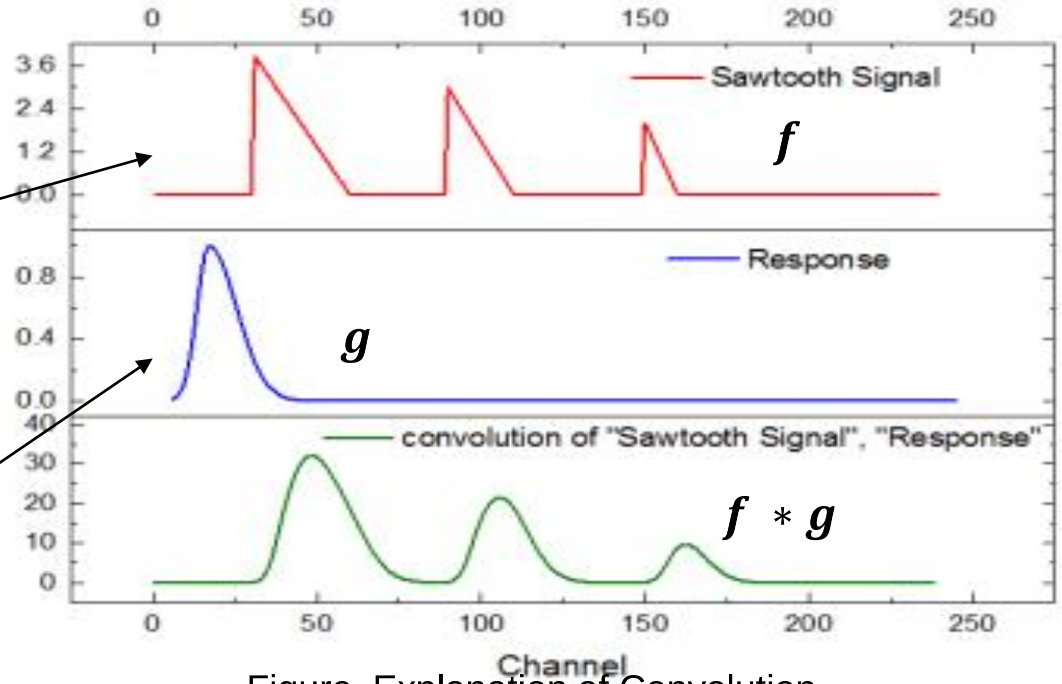


Figure. Explanation of Convolution.

https://images.app.goo.gl/BnStzc94Mb3QWsUA9

The convolution of f and g is written f * g , denoting the operator with the symbol * . It is defined as the integral of the product of the two functions after one is reflected about the y-axis and shifted. The integral is evaluated for all values of shift, producing the convolution function.

$$f * g = f * g = \int_{-\infty}^{+\infty} f(\tau)g(t-\tau)d\tau$$

2.5 Consider a real beam with limited size

Convolution to get dose distribution from a real beam

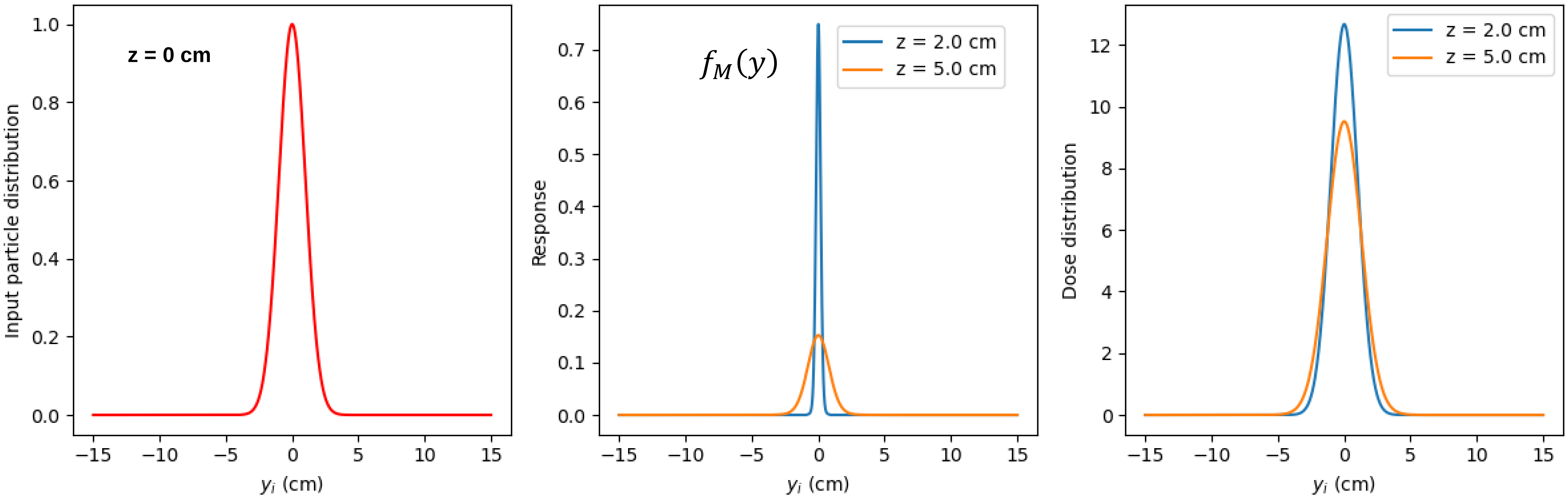


Fig. Convolution to get Dose Distribution

3. Benchmark with FLUKA Simulation

Percentage Depth Dose (PDD) in water

- Input beam:
  - Beam energy 18 MeV
  - RMS size 0.065 cm
  - Gaussian distribution
  - Parallel beam

- Computation time: ~seconds

Percentage Depth Dose(PDD): describes the on-axis dose along the water

1. Calculate the dose distribution (vs x and y) at each depth (z)
2. Get the dose at the z-axis (x=y=0)

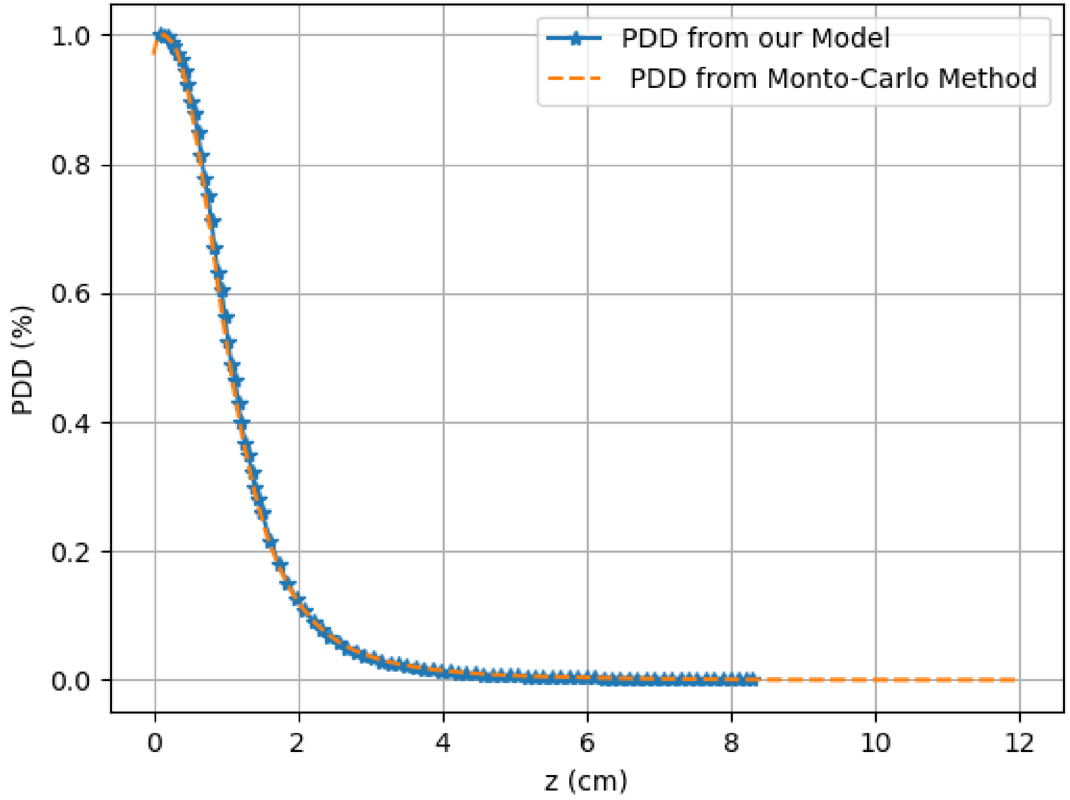


Fig. Comparison of results from 2 different approaches.

Thanks to Zohrab for FLUKA simulation!

4. Conclusion

- A python script has been developed for Fast estimation of dose distribution generated by electron beam based on Moliere theory.
- This script gives results with same accuracy of MC code, but with much shorter computing time.

Further developments:

- Consider scattering angles at the entrance
- This model can be used for mediums other than water like air, Al etc.

Applications:

- Can be adapted for more complex setup like FLASHlab@PITZ for online dose determination.
- Can be used to optimize treatment plan in FLASH RT

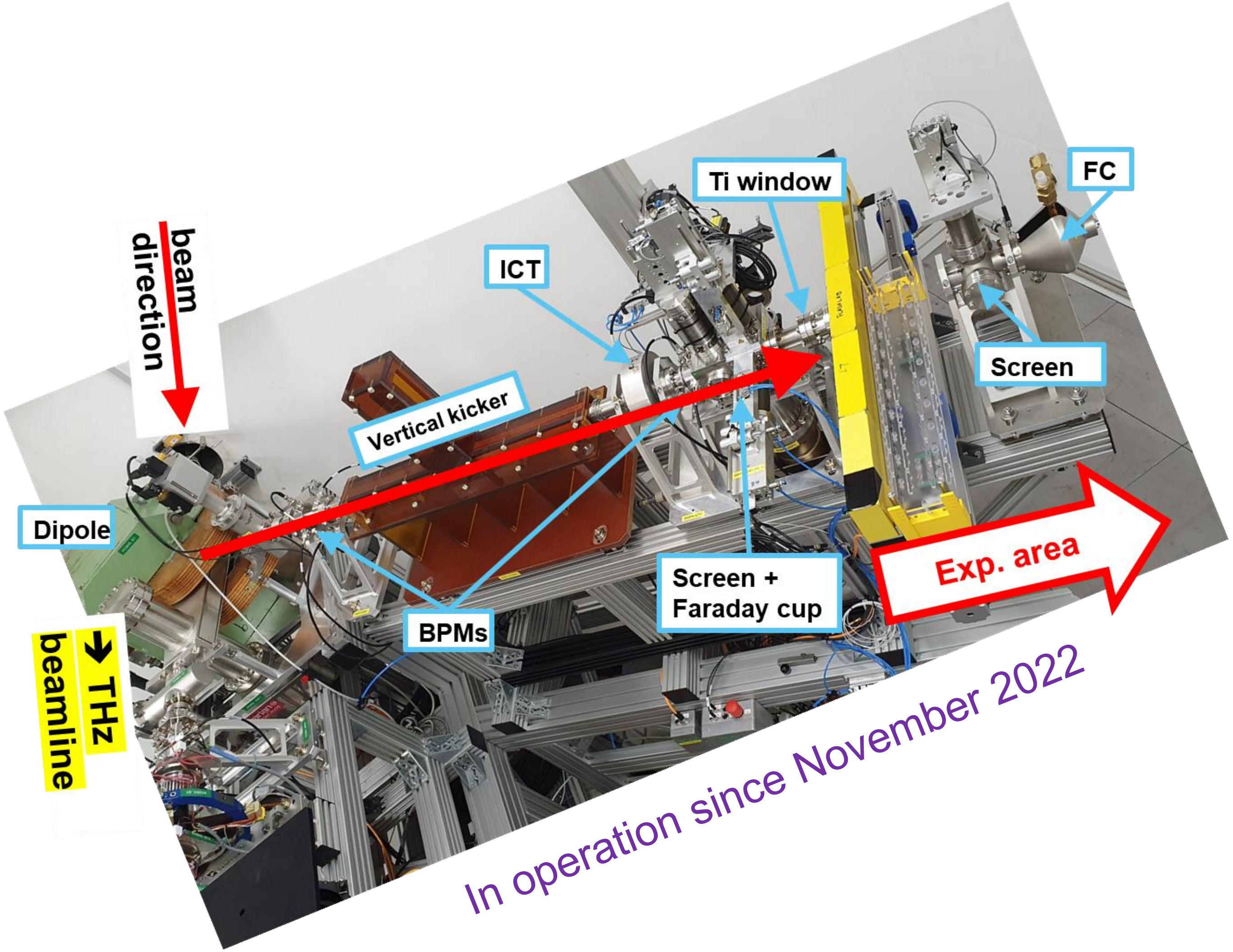

THANK YOU!

References

- Molière 1948
- https://de.wikipedia.org/wiki/Monte-Carlo-Simulation
- https://iopscience.iop.org/article/10.1088/0031-9155/61/4/N102/pdf

Background

Startup Beamline for FLASHlab@PITZ



Beam Direction – shows the direction in which the particle beam travel through the setup.
THz beamline – indicates the path taken by the terahertz radiation, which is often used in diagnosis.
Dipole – used to bend path of electrons in the beam. It can also be used to separate particles based on their momentum.
BPMs – used to measure the position of the particle beam along the beamline. They provide feedback to ensure the beam is correctly aligned.
Vertical kicker – used to deflect the beam vertically.
ICT – Integrating current transformer – used to measure the beam current . It integrates the current over time to provide a measure of the total charge in beam.